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TABLES OF TRANSMISSION LINE CONSTANTS

BY
D. D. EWING

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ENGINEERING EXPERIMENT STATION
LAFAYETTE, INDIANA

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TABLES OF TRANSMISSION LINE CONSTANTS

BY

D. D. EWING
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PURDUE UNIVERSITY
LAFAYETTE, INDIANA
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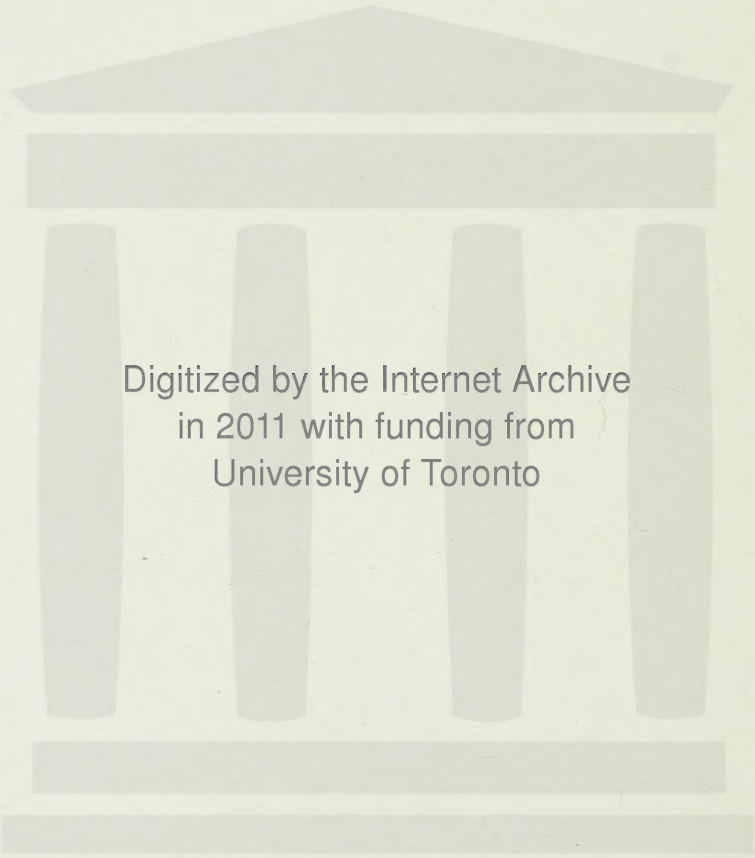
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TABLE OF CONTENTS

Page

Object 5
Exact Formulae for Transmission Line Voltages and Currents 5
Explanation of the Tables 6
Application of the Tables 6

APPENDICES

Appendix A—
Derivation of Exact Formulae for Transmission Line Currents and Voltages 17
Appendix B—
Calculation of Inductive Reactances 22
Appendix C—
Calculation of Shunted Admittances 26
Appendix D—
Sample Calculations of the Tables 30

LIST OF TABLES

Table 1 Showing the Range of Table 2 and 3 9
Table 2 10
Propagation Constants and Surge Resistances for 60 Cycle Lines.
Table 3 13
Inductive Reactances and Shunted Admittances for 60 Cycle Lines
Table 4 16
Cable Diameters and Resistance per Mile

LIST OF FIGURES

Fig. 1—Simplified Line Diagram 17
Fig. 2—Electromagnetic Field Surrounding a Conductor ... 22
Fig. 3—Electromagnetic Field Within a Conductor 23
Fig. 4—Electrostatic Field Surrounding a Conductor 27

TABLES OF TRANSMISSION LINE CONSTANTS

The object of this Bulletin is to present tables of certain line constants which will be of assistance in the application of exact formulae to the solution of electric transmission line problems.

The laborious work of calculating the numerical values set forth in the tables, was performed by D. L. Bement and L. M. Clearwaters as their senior thesis in Electrical Engineering, 1922. The calculations have been checked by S. H. Ahmed, graduate student in Electrical Engineering 1922–23.

The development of suitable formulae for transmission line calculations has received a great deal of attention from engineers and physicists. With this development are linked the names of Heaviside, Fleming, Steinmetz, Kennelly, Karapetoff, and others. To Kennelly is due the development of the methods of hyperbolic trigonometry.

Exact formulae for the computation of transmission line voltages and currents are as follows:

$$\dot{E}_g = \dot{E}_r \cosh L \sqrt{zy_s} + \dot{I}_r \sqrt{\frac{z}{y_s}} \sinh L \sqrt{zy_s}$$

$$\dot{E} = \dot{E}_r \cosh L_2 \sqrt{zy_s} + \dot{I}_r \sqrt{\frac{z}{y_s}} \sinh L_2 \sqrt{zy_s}$$

$$\dot{I}_g = \dot{I}_r \cosh L \sqrt{zy_s} + \dot{E}_r \sqrt{\frac{y_s}{z}} \sinh L \sqrt{zy_s}$$

$$\dot{I} = \dot{I}_r \cosh L_2 \sqrt{zy_s} + \dot{E}_r \sqrt{\frac{y_s}{z}} \sinh L_2 \sqrt{zy_s}$$

In these formulae all quantities, except distance, are complex numbers and:

E_g is the voltage at the generator end of the transmission line.
I_g is the current at the generator end of the transmission line.
L is the length of the transmission line.
L_2 is the distance from load end to any point P.
I is the current at the point P.
E is the voltage at the point P.

$\sqrt{zy_s}$ is known as the propagation constant or hyperbolic angle of the line.

$\sqrt{\frac{z}{y_s}}$ is called the surge impedance of the line.

In this Bulletin, tables of $\sqrt{zy_s}$ and $\sqrt{\frac{z}{y_s}}$ for 60 cycles and for the range of conductor sizes and spacings shown in Table 1, are presented.

Values are presented in Table 2 for the propagation constants in hyperbolic radians per mile, and the surge impedances in ohms per mile of conductor. In the computation of Table 2 it was necessary to calculate the inductive reactances and shunted admittances per mile of conductor which are shown in Table 3. In making these calculations the outside diameter of the cable was used as the diameter of the conductor. Cable diameters and the corresponding resistances at 25° Centigrade per mile of conductor are set forth in Table 4.

Tables 2, 3 and 4 are computed for standard, annealed copper, concentric lay cable. The range of spacings, 24 inches to 216 inches, and the cable sizes, No. 0.A.W.G., to 750,000 circular mils, cover the range commonly used in electric power transmission lines. Table 1 shows diagrammatically the range of values covered by Tables 2 and 3. The star indicates the spacings and conductor sizes for which propagation constants, surge impedances, inductive reactances and shunted admittances may be found in Tables 2 and 3.

In applying values from Table 2 in the fundamental exact formulae it is necessary to use the hyperbolic sine and cosine of the propagation constant. These may be obtained from Dr. A. E. Kennelly's "Tables of Complex Hyperbolic and Circular Functions," or obtained graphically from the "Chart Atlas of Hyperbolic and Circular Functions" published by Dr. Kennelly as a supplement to his "Tables".

In the exact formulae all electrical quantities are written as vectors expressed in either polar or rectangular coordinates.

An example of the use of the tables follows.

APPLICATION OF THE TABLES

Given: A 3 phase electric power transmission line 241 miles long: Equidistant triangular spacing, 3 wires 216″ apart: Cable size, 600,000 circular mils; Line to line voltage at the load, 150,000 volts: Load, 70,000 KVA. at .8 power factor.

Required: Voltage to neutral and the current per conductor at the generator end of the line.

The exact formulae for the voltage and current at the generator are:

$$\dot{E}_g = \dot{E}_r \cosh L \sqrt{zy_s} + \dot{I}_r \sqrt{\frac{z}{y_s}} \sinh L \sqrt{zy_s}$$

$$\dot{I}_g = \dot{I}_r \cosh L \sqrt{zy_s} + \dot{E}_r \sqrt{\frac{y}{z}} \sinh L \sqrt{zy_s}$$

The calculation of voltage at the generator is as follows:

$$E_r = \frac{150{,}000}{1.732} = 86{,}666 \text{ volts, line to neutral.}$$

Taking the line to neutral voltage as a reference line,

$$\dot{E}_r = 86{,}666 \ (1 + j\ 0) = (86{,}666 + j\ 0)$$

$$I_r = \frac{70{,}000{,}000}{3 \times 86.666} = 269.5 \text{ amperes}$$

$$\dot{I}_r = 269.5 \ (.8 - j\ .6) = (215.6 - j\ 161.7)$$

$$\dot{E}_g = (86{,}666 + j\ 0) \cosh 241 \sqrt{zy_s} + (215.6 - j\ 161.7) \sqrt{\frac{z}{y_s}} \sinh 241 \sqrt{zy_s}$$

The values for the $\sqrt{zy_s}$ and $\sqrt{\frac{z}{y_s}}$ may be found from Table 2. Values taken directly from the table must be multiplied by the inverse of the multiplier accompanying $\sqrt{zy_s}$ or $\sqrt{\frac{z}{y_s}}$ respectively.

Entering Table 2, for cable size 600,000 circular mils and spacing 216″ we find

$$\sqrt{zy_s} \times 10^3 = (.1251 + j\ 2.065)$$

$$\text{or } \sqrt{zy_s} = (.1251 + j\ 2.065) \times 10^{-3}$$

$$\sqrt{\frac{z}{y_s}} \times 10^{-3} = (.3790 - j\ .0230)$$

$$\text{or } \sqrt{\frac{z}{y_s}} = (.3790 - j\ .0230) \times 10^3.$$

Substituting these values in the equation for $\dot{E}_g$

$$\dot{E}_g = (86{,}666 + j\ 0) \cosh [241\ (.1251 + j\ 2.065) \times 10^{-3}] + (215.6 - j\ 161.7)\ (.3790 - j\ .0230) \times 10^3 \times \sinh [241\ (.1251 + j\ 2.065) \times 10^{-3}],$$

or .

$$\dot{E}_g = (86{,}666 + j\,0)\ \cosh\ (.0302 + j\ .4952) + 10^3\ (77.89 - j\ 66.15)\ \sinh\ (.0302 + j\ .4952)$$

Cosh $\sqrt{zy_s}$ and *sinh* $\sqrt{zy_s}$ may be found from the tables of complex hyperbolic and circular functions, *cosh* $\sqrt{zy_s}$, equalling $.710 + j\ .204$ and *sinh* $\sqrt{zy_s}$, $.0216 + j\ .700$.
Therefore,

$$\dot{E}_g = (86{,}666 + j\,0)\ (.710 + j\ .204) + (77{,}890 - j\ 66{,}150)\ (.0216 + j\ .700)$$

Simplifying this expression,

$$\dot{E}_g = (109517 + j\ 53100)$$

or $E_g = 121{,}600$ volts, line to neutral.

Similarly the calculation of current at the generator is: Substituting in the formula for I_g the various values used in computing $\dot{E}_g$

$$\dot{I}_g = (215.6 - j\ 161.7)\ \cosh\ (.0302 + j\ .4952) + \frac{(86{,}666 + j\,0)}{(.3790 - j\ .0230) \times 10^3}\ (\sinh\ (.0302 + j\ .4952)$$

$$= (215.6 - j\ 161.7)\ (.710 + j\ .204) + \frac{(86{,}666 + j\,0)\ (.0216 + j\ .700)}{(.3790 - j\ .0230) \times 10^3}$$

Simplifying

$$\dot{I}_g = (185.4 - j\ 48.2)$$

or $I = 191$ amperes per conductor.

As the surge impedance and propagation constant enter into many transmission lines problems other than those of voltages and currents the tables have a much wider application than might be inferred from the single example here illustrated.

TABLE 1.

Range of the Computed Values.

Spacing in Inches

Cable Size	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
0	*	*	*	*	*	*	*										
00	*	*	*	*	*	*	*	*									
000	*	*	*	*	*	*	*	*	*								
0000	*	*	*	*	*	*	*	*	*	*							
250,000	*	*	*	*	*	*	*	*	*	*	*						
300,000	*	*	*	*	*	*	*	*	*	*	*	*					
350,000	*	*	*	*	*	*	*	*	*	*	*	*	*				
400,000		*	*	*	*	*	*	*	*	*	*	*	*	*			
450,000			*	*	*	*	*	*	*	*	*	*	*	*	*		
500,000				*	*	*	*	*	*	*	*	*	*	*	*	*	
550,000					*	*	*	*	*	*	*	*	*	*	*	*	*
600,000						*	*	*	*	*	*	*	*	*	*	*	*
650,000							*	*	*	*	*	*	*	*	*	*	*
700,000								*	*	*	*	*	*	*	*	*	*
750,000									*	*	*	*	*	*	*	*	*

TABLE 2.

Propagation Constants and Surge Resistances for 60 Cycle Lines.

Wire Size A.W.G. or Circular Mils	Spacing —24 inches		Spacing—36 inches	
	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0	(.8320 +j 2.230)	(.4060 −j .1517)	(.7780 +j 2.205)	(.3455 −j .1219)
00	(.6950 +j 2.185)	(.3077 −j .0977)	(.6470 +j 2.165)	(.3305 −j .0986)
000	(.5735 +j 2.158)	(.2955 −j .0786)	(.5110 +j 2.140)	(.3204 −j .0767)
0000	(.4720 +j 2.130)	(.2847 −j .1632)	(.4350 +j 2.113)	(.3088 −j .0635)
250,000	(.4093 +j 2.123)	(.2782 −j .0537)	(.3760 +j 2.103)	(.3017 −j .0539)
300,000	(.3505 +j 2.112)	(.2715 −j .0450)	(.3592 +j 2.094)	(.2940 −j .0504)
350,000	(.3068 +j 2.102)	(.2645 −j .0386)	(.2798 +j 2.090)	(.2899 −j .0387)
400,000			(.2492 +j 2.088)	(.2862 −j .0341)
450,000				
500,000				
550,000				
600,000				
650,000				
700,000				
750,000				

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—48 inches		Spacing—60 inches	
	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0	(.7420 +j 2.193)	(.3619 −j .1223)	(.7150 +j 2.181)	(.3750 −j .1290)
00	(.6160 +j 2.160)	(.3480 −j .0993)	(.5940 +j 2.146)	(.3605 −j .0997)
000	(.5025 +j 2.125)	(.3358 −j .0794)	(.4850 +j 2.120)	(.3496 −j .0799)
0000	(.4120 +j 2.108)	(.3262 −j .0638)	(.3965 +j 2.101)	(.3385 −j .0639)
250,000	(.3560 +j 2.100)	(.3195 −j .0542)	(.3426 +j 2.096)	(.3320 −j .0542)
300,000	(.3039 +j 2.090)	(.3122 −j .0453)	(.2918 +j 2.088)	(.3261—j .0455)
350,000	(.2659 +j 2.085)	(.3062 −j .0390)	(.2540 +j 2.083)	(.3195 −j .0389)
400,000	(.2360 +j 2.083)	(.3019 −j .0342)	(.2255 +j 2.078)	(.3148 −j .0342)
450,000	(.2118 +j 2.082)	(.2985 −j .0304)	(.2030 +j 2.078)	(.3110 −j .0304)
500,000			(.1852 +j 2.078)	(.3080 −j .0274)
550,000				
600,000				
650,000				
700,000				
750,000				

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—72 inches		Spacing—84 inches	
	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0	(.7010 +j 2.178)	(.3845 −j .1240)	(.6830 +j 2.165)	(.3930 −j .1240)
00	(.5760 +j 2.141)	(.3712 −j .0998)	(.5620 +j 2.135)	(.3805 −j .1001)
000	(.4705 +j 2.116)	(.3600 −j .0801)	(.4590 +j 2.100)	(.3670 −j .0801)
0000	(.3833 +j 2.097)	(.3490 −j .0638)	(.3733 +j 2.095)	(.3590 −j .0640)
250,000	(.3310 +j 2.088)	(.3425 −j .0543)	(.3220 +j 2.084)	(.3521 −j .0543)
300,000	(.2826 +j 2.086)	(.3360 −j .0456)	(.2740 +j 2.081)	(.3455 −j .0455)
350,000	(.2451 +j 2.081)	(.3308 −j .0390)	(.2385 +j 2.078)	(.3395 −j .0390)
400,000	(.2180 +j 2.080)	(.3260 −j .0342)	(.2119 +j 2.078)	(.3352 −j .0342)
450,000	(.1966 +j 2.079)	(.3221 −j .0304)	(.1906 +j 2.076)	(.3320 −j .0305)
500,000	(.1783 +j 2.078)	(.3195 −j .0274)	(.1741 +j 2.074)	(.3280 −j .0275)
550,000	(.1632 +j 2.075)	(.3160 −j .0248)	(.1587 +j 2.072)	(.3250 −j .0249)
600,000			(.1471 +j 2.072)	(.3222 −j .0229)
650,000				
700,000				
750,000				

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—96 inches		Spacing—108 inches	
	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0	(.6700 +j 2.172)	(.4020 −j .1240)		
00	(.5525 +j 2.137)	(.3882 −j .1002)	(.5415 +j 2.122)	(.3950 −j .1005)
000	(.4500 +j 2.132)	(.3765 −j .0804)	(.4405 +j 2.101)	(.3830 −j .0803)
0000	(.3657 +j 2.100)	(.3670 −j .0640)	(.3590 +j 2.085)	(.3730 −j .0641)
250,000	(.3158 +j 2.090)	(.3605 −j .0544)	(.3090 +j 2.078)	(.3662 −j .0544)
300,000	(.2680 +j 2.080)	(.3535 −j .0455)	(.2632 +j 2.077)	(.3604 −j .0456)
350,000	(.2335 +j 2.075)	(.3480 −j .0381)	(.2282 +j 2.076)	(.3548 −j .0391)
400,000	(.2072 +j 2.075)	(.3440 −j .0343)	(.2038 +j 2.075)	(.3505 −j .0344)
450,000	(.1863 +j 2.075)	(.3400 −j .0305)	(.1822 +j 2.074)	(.3468 −j .0305)
500,000	(.1698 +j 2.075)	(.3360 −j .0275)	(.1659 +j 2.073)	(.3438 −j .0275)
550,000	(.1551 +j 2.073)	(.3330 −j .0249)	(.1510 +j 2.073)	(.3408 −j .0248)
600,000	(.1445 +j 2.073)	(.3301 −j .0230)	(.1396 +j 2.071)	(.3380 −j .0228)
650,000	(.1335 +j 2.073)	(.3279 −j .0211)	(.1310 +j 2.071)	(.3355 −j .0212)
700,000			(.1219 +j 2.071)	(.3326 −j .0196)
750,000				

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—120 inches		Spacing—132 inches	
	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0				
00				
000	(.4350 +j 2.103)	(.3939 −j .0806)		
0000	(.3539 +j 2.097)	(.3800 −j .0642)	(.3480 +j 2.085)	(.3855 −j .0643)
250,000	(.3043 +j 2.080)	(.3725 −j .0545)	(.2995 +j 2.082)	(.3790 −j .0545)
300,000	(.2590 +j 2.079)	(.3672 −j .0457)	(.2548 +j 2.077)	(.3722 −j .0457)
350,000	(.2248 +j 2.078)	(.3617 −j .0391)	(.2210 +j 2.071)	(.3670 −j .0392)
400,000	(.1995 +j 2.076)	(.3566 −j .0343)	(.1960 +j 2.070)	(.3622 −j .0343)
450,000	(.1797 +j 2.075)	(.3525 −j .0305)	(.1761 +j 2.070)	(.3580 −j .0305)
500,000	(.1632 +j 2.072)	(.3500 −j .0276)	(.1598 +j 2.069)	(.3551 −j .0274)
550,000	(.1490 +j 2.072)	(.3470 −j .0250)	(.1469 +j 2.068)	(.3524 −j .0250)
600,000	(.1382 +j 2.072)	(.3440 −j .0230)	(.1352 +j 2.067)	(.3498 −j .0229)
650,000	(.1285 +j 2.072)	(.3429 −j .0212)	(.1263 +j 2.066)	(.3470 −j .0212)
700,000	(.1201 +j 2.072)	(.3382 −j .0197)	(.1180 +j 2.066)	(.3442 −j .0197)
750,000	(.1129 +j 2.070)	(.3375 −j .0184)	(.1107 +j 2.065)	(.3430 −j .0184)

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—144 inches		Spacing—156 inches	
	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{zy_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0				
00				
000				
0000				
250,000	(.2955 +j 2.078)	(.3838 −j .0545)		
300,000	(.2519 +j 2.075)	(.3775 −j .0458)	(.2479 +j 2.075)	(.3820 −j .0456)
350,000	(.2176 +j 2.072)	(.3720 −j .0391)	(.2154 +j 2.072)	(.3766 −j .0392)
400,000	(.1930 +j 2.071)	(.3680 −j .0343)	(.1912 +j 2.066)	(.3710 −j .0344)
450,000	(.1739 +j 2.069)	(.3634 −j .0305)	(.1714 +j 2.065)	(.3690 −j .0306)
500,000	(.1578 +j 2.068)	(.3609 −j .0276)	(.1556 +j 2.065)	(.3660 −j .0275)
550,000	(.1447 +j 2.068)	(.3580 −j .0250)	(.1420 +j 2.064)	(.3627 −j .0250)
600,000	(.1335 +j 2.067)	(.3550 −j .0229)	(.1319 +j 2.064)	(.3600 −j .0230)
650,000	(.1245 +j 2.067)	(.3520 −j .0213)	(.1222 +j 2.064)	(.3579 −j .0212)
700,000	(.1167 +j 2.066)	(.3490 −j .0197)	(.1142 +j 2.064)	(.3550 −j .0187)
750,000	(.1093 +j 2.065)	(.3474 −j .0184)	(.1081 +j 2.064)	(.3525 −j .0185)

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—168 inches		Spacing—180 inches	
	$\sqrt{z y_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{z y_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0				
00				
000				
0000				
250,000				
300,000				
350,000	(.2122 +j 2.080)	(.3825 −j .0386)		
400,000	(.1920 +j 2.075)	(.3778 −j .0350)	(.1872 +j 2.069)	(.3812 −j .0345)
450,000	(.1699 +j 2.073)	(.3725 −j .0305)	(.1674 +j 2.066)	(.3775 −j .0306)
500,000	(.1547 +j 2.073)	(.3698 −j .0276)	(.1522 +j 2.066)	(.3742 −j .0276)
550,000	(.1410 +j 2.070)	(.3362 −j .0250)	(.1392 +j 2.064)	(.3710 −j .0250)
600,000	(.1309 +j 2.068)	(.3639 −j .0232)	(.1292 +j 2.064)	(.3680 −j .0231)
650,000	(.1215 +j 2.065)	(.3614 −j .0213)	(.1201 +j 2.063)	(.3660 −j .0213)
700,000	(.1128 +j 2.065)	(.3600 −j .0196)	(.1119 +j 2.062)	(.3635 −j .0197)
750,000	(.1060 +j 2.065)	(.3575 −j .0184)	(.1051 +j 2.062)	(.3605 −j .0184)

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—192 inches		Spacing 204 inches	
	$\sqrt{z y_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{z y_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}
0				
00				
000				
0000				
250,000				
300,000				
350,000				
400,000				
450,000	(.1658 +j 2.067)	(.3808 −j .0305)		
500,000	(.1508 +j 2.065)	(.3780 −j .0275)	(.1492 +j 2.065)	(.3820 −j .0276)
550,000	(.1382 +j 2.064)	(.3740 −j .0251)	(.1368 +j 2.065)	(.3780 −j .0250)
600,000	(.1276 +j 2.064)	(.3719 −j .0230)	(.1262 +j 2.065)	(.3755 −j .0230)
650,000	(.1192 +j 2.064)	(.3690 −j .0213)	(.1173 −j 2.065)	(.3738 −j .0212)
700,000	(.1107 +j 2.063)	(.3670 −j .0197)	(.1093 +j 2.065)	(.3705 −j .0197)
750,000	(.1040 +j 2.063)	(.3660 −j .0185)	(.1035 +j 2.065)	(.3683 −j .0185)

TABLE 2—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—216 inches		Spacing	
	$\sqrt{z y_s}$ x 10^3	$\sqrt{z/y_s}$ x 10^{-3}	$\sqrt{z y_s}$	$\sqrt{z/y_s}$
0				
00				
000				
0000				
250,000				
300,000				
350,000				
400,000				
450,000				
500,000				
550,000	(.1349 +j 2.065)	(.3820 −j .0249)		
600,000	(.1251 +j 2.065)	(.3790 −j .0230)		
650,000	(.1160 +j 2.065)	(.3770 −j .0212)		
700,000	(.1082 +j 2.065)	(.3740 −j .0191)		
750,000	(.1021 +j 2.065)	(.3719 −j .0184)		

TABLE 3.

Inductive Reactances and Shunted Admittances for 60 Cycle Lines.

Wire Size A.W.G. or Circular Mils	Spacing—24 inches		Spacing—36 inches	
	x	y_s	x	y_s
0	.620	6.92 x 10^{-6}	.668	6.39 x 10^{-6}
00	.605	7.11	.654	6.55
000	.591	7.28	.641	6.69
0000	.576	7.47	.625	6.85
250,000	.567	7.62	.615	6.98
300,000	.556	7.78	.604	7.12
350,000	.546	7.94	.596	7.23
400,000			.590	7.30
450,000				
500,000				
550,000				
600,000				
650,000				
700,000				
750,000				

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—48 inches		Spacing—60 inches	
	x	y_s	x	y_s
0	.703	6.07 x 10^{-6}	.730	5.83 x 10^{-6}
00	.690	6.20	.716	5.96
000	.675	6.33	.703	6.07
0000	.661	6.48	.687	6.21
250,000	.651	6.58	.678	6.31
300,000	.640	6.70	.667	6.42
350,000	.630	6.81	.657	6.52
400,000	.621	6.91	.648	6.60
450,000	.615	6.98	.641	6.69
500,000			.635	6.75
550,000				
600,000				
650,000				
700,000				
750,000				

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—72 inches		Spacing—84 inches	
	x	y_s	x	y_s
0	.752	5.65 x 10^{-6}	.770	5.51 x 10^{-6}
00	.737	5.77	.757	5.61
000	.724	5.88	.742	5.72
0000	.710	6.01	.730	5.84
250,000	.699	6.10	.719	5.93
300,000	.688	6.21	.708	6.03
350,000	.680	6.29	.698	6.12
400,000	.671	6.38	.691	6.20
450,000	.663	6.45	.682	6.26
500,000	.658	6.51	.676	6.33
550,000	.652	6.57	.671	6.38
600,000			.665	6.44
650,000				
700,000				
750,000				

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—96 inches		Spacing—108 inches	
	x	y_s	x	y_s
0	.787	5.40 x 10^{-6}	----	----
00	.774	5.50	.787	5.39 x 10^{-6}
000	.758	5.60	.773	5.49
0000	.745	5.71	.759	5.60
250,000	.735	5.80	.748	5.69
300,000	.724	5.88	.738	5.77
350,000	.714	5.97	.728	5.84
400,000	.708	6.04	.720	5.92
450,000	.698	6.11	.713	5.98
500,000	.692	6.17	.707	6.03
550,000	.686	6.22	.702	6.08
600,000	.681	6.28	.697	6.13
650,000	.676	6.33	.691	6.18
700,000	----	----	.687	6.22
750,000	----	----	----	----

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—120 inches		Spacing—132 inches	
	x	y_s	x	y_s
0	----	----	----	----
00	----	----	----	----
000	.786	5.40 x 10^{-6}	----	----
0000	.771	5.50	.783	5.42 x 10^{-6}
250,000	.759	5.58	.773	5.50
300,000	.751	5.66	.762	5.58
350,000	.741	5.74	.752	5.65
400,000	.733	5.81	.744	5.71
450,000	.726	5.87	.737	5.77
500,000	.720	5.92	.731	5.82
550,000	.714	5.97	.725	5.87
600,000	.708	6.02	.720	5.92
650,000	.703	6.07	.715	5.96
700,000	.699	6.11	.710	6.00
750,000	.695	6.14	.707	6.03

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—144 inches		Spacing—156 inches	
	x	y_s	x	y_s
0	----	----	----	----
00	----	----	----	----
000	----	----	----	----
0000	----	----	----	----
250,000	.783	5.41 x 10^{-6}	----	----
300,000	.772	5.50	.782	5.43 x 10^{-6}
350,000	.763	5.57	.773	5.50
400,000	.756	5.64	.760	5.56
450,000	.748	5.70	.756	5.61
500,000	.741	5.74	.752	5.66
550,000	.736	5.79	.746	5.70
600,000	.731	5.83	.741	5.74
650,000	.725	5.87	.736	5.78
700,000	.720	5.91	.731	5.83
750,000	.717	5.95	.726	5.87

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—168 inches		Spacing—180 inches	
	x	y_s	x	y_s
0				
00				
000				
0000				
250,000				
300,000				
350,000	.785	5.43 x 10-6	.782	5.43 x 10-6
400,000	.775	5.48	.775	5.48
450,000	.776	5.55	.769	5.52
500,000	.760	5.60	.763	5.57
550,000	.754	5.65	.758	5.61
600,000	.749	5.68	.752	5.65
650,000	.744	5.72	.748	5.69
700,000	.741	5.75	.743	5.73
750,000	.736	5.79		

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—192 inches		Spacing—204 inches	
	x	y_s	x	y_s
0				
00				
000				
0000				
250,000				
300,000				
350,000				
400,000				
450,000	.783	5.43 x 10-6	.785	5.42 x 10-6
500,000	.777	5.47	.778	5.47
550,000	.770	5.52	.773	5.50
600,000	.765	5.56	.769	5.53
650,000	.760	5.60	.764	5.57
700,000	.756	5.63	.759	5.60
750,000	.752	5.66		

TABLE 3—*Continued.*

Wire Size A.W.G. or Circular Mils	Spacing—216 inches		Spacing—	
	x	y_s	x	y_s
0				
00				
000				
0000				
250,000				
300,000				
350,000				
400,000				
450,000				
500,000				
550,000	.785	5.40 x 10-6		
600,000	.780	5.45		
650,000	.775	5.48		
700,000	.770	5.52		
750,000	.765	5.56		

TABLE 4.

Resistance per Mile of Standard Concentric-Lay Annealed Copper Cable at 25° C.

Cable Size A.W.G. or Circular Mils	Outside Diameter of Cable Inches	Resistance per Mile of Cable
0	.373	.5380
00	.418	.4280
000	.470	.3386
0000	.528	.2685
250,000	.575	.2275
300,000	.630	.1900
350,000	.681	.1625
400,000	.728	.1424
450,000	.772	.1267
500,000	.814	.1140
550,000	.855	.1033
600,000	.893	.0950
650,000	.929	.0876
700,000	.964	.0813
750,000	.998	.0760

APPENDIX A

DERIVATION OF EXACT FORMULAE FOR THE COMPUTATION OF TRANSMISSION LINE VOLTAGES AND CURRENTS

The exact formulae for computation of transmission line voltages and currents may be derived as follows:

Given the electric power transmission line represented by Figure 1, L units long, with a differential section dl, l units from the load end of the line.

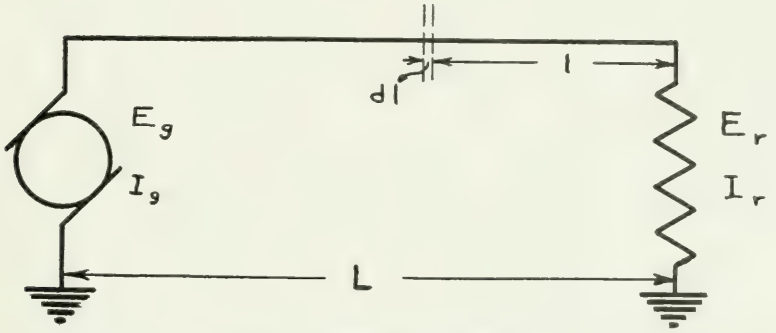

Fig. 1. Simplified Line Diagram.

Let

E_r = the voltage line to ground at the load end,
I_r = the current in the wire at the load end,
E_g = the voltage line to ground at the generator,
I_g = the current in the wire at the generator,
E = the voltage line to ground at the point distant l units from the load end,
I = the current in the wire at the point distant l units from the load end.

The fall of potential over the section dl is

$$dE = I\ (R + jx)\ dl \qquad (1)$$

where R is the resistance per unit length of line and x is the inductive reactance per unit length of line, both being constants.

The drop in the current value in the section dl is

$$\dot{dI} = \dot{E} y_s \, dl \qquad (2)$$

where y_s is the shunted admittance per unit length of line, a constant.

Dividing equation (1) through by dl and substituting z for its equal $(R+jx)$

$$\frac{\dot{dE}}{dl} = \dot{I} \, (R+jx) = \dot{I}z \qquad (3)$$

in which z is a constant.

Dividing equation (2) by dl

$$\frac{\dot{dI}}{dl} = \dot{E} y_s$$

in which y_s is a constant. (4)

Differentiating equation (3) and dividing by dl,

$$\frac{\dot{d^2E}}{dl^2} = z \frac{\dot{dI}}{dl} \qquad (5)$$

Differentiating equation (4) and dividing by dl,

$$\frac{\dot{d^2I}}{dl^2} = y_s \frac{\dot{dE}}{dl} \qquad (6)$$

Substituting equation (4) in equation (5)
and equation (3) in equation (6),

$$\frac{\dot{d^2E}}{dl^2} = \dot{E} \, z \, y_s, \qquad (7)$$

and

$$\frac{\dot{d^2I}}{dl^2} = \dot{I} \, z \, y_s, \qquad (8)$$

where the product zy_s is a constant.

Solution of the differential equation (7) gives

$$\dot{E} = A\epsilon^{\sqrt{zy_s}\, l} + B\epsilon^{-\sqrt{zy_s}\, l} \qquad (9)$$

in which A and B are constants whose values are to be evaluated

(In equation (9) "ϵ" is the base of the Naperian system of logarithms and is equal to *2.7183*)

Since $\frac{dE}{dl} = Iz,$ (3)

$$I = \frac{dE}{z\ dl}$$

Taking the first derivative of equation (9) with respect to l

$$I = \frac{\sqrt{zy_s}\ A\ \epsilon^{\sqrt{zy_s}\ l} - \sqrt{zy_s}\ B\ \epsilon^{-\sqrt{zy_s}\ l}}{z} \quad (10)$$

or

$$I = \sqrt{\frac{y_s}{z}}\ A\ \epsilon^{\sqrt{zy_s}\ l} - \sqrt{\frac{y_s}{z}}\ B\epsilon^{-\sqrt{zy_s}\ l}, \quad (11)$$

$$= \sqrt{\frac{y_s}{z}}\ (A\epsilon^{\sqrt{zy_s}\ l} - B\epsilon^{-\sqrt{zy_s}\ l})$$

A and B are constants depending on the terminal conditions of the line.

When $l = O,$

$$E = E_r,$$

and $I = I_r.$

Substituting these values in equations (9) and (11)

$$E_r = (A + B), \quad (12)$$

and $$I_r = \sqrt{\frac{y_s}{z}}\ (A - B) \quad (13)$$

When $l = L,$

$$E = E_g,$$

and $I = I_g.$

Substituting these values in equation (*9*) and (*11*)

$$\dot{E}_g = A\epsilon^{\sqrt{zy_s}\,L} + B\epsilon^{-\sqrt{zy_s}\,L}, \tag{14}$$

and

$$\dot{I}_g = \sqrt{\frac{y_s}{z}}\left(A\epsilon^{\sqrt{zy_s}\,L} - B\epsilon^{-\sqrt{zy_s}\,L}\right) \tag{15}$$

But, $\epsilon^{\theta} = \cosh\ \theta + \sinh\ \theta,$

and, $\epsilon^{-\theta} = \cosh\ \theta - \sinh\ \theta,$

Therefore, $\epsilon^{\sqrt{zy_s}\,L} = \cosh L\sqrt{zy_s} + \sinh L\sqrt{zy_s},$ (16)

and, $\epsilon^{-\sqrt{zy_s}\,L} = \cosh L\sqrt{zy_s} - \sinh L\sqrt{zy_s}.$ (17)

Substituting equations (*16*) and (*17*) in (*14*)

$$\dot{E}_g = A\ (\cosh L\sqrt{zy_s} + \sinh L\sqrt{zy_s}) + B\ (\cosh L\sqrt{zy_s} - \sinh L\sqrt{zy_s}),$$

or, $\dot{E}_g = (A+B)\cosh L\sqrt{zy_s} + (A-B)\sinh L\sqrt{zy_s}$ (18)

Substituting the values for $(A+B)$ and $(A-B)$ from equations (*12*) and (*13*) in equation (*18*),

$$\dot{E}_g = \dot{E}_r \cosh L\sqrt{zy_s} + \dot{I}_r\sqrt{\frac{z}{y_s}}\sinh L\sqrt{zy_s} \tag{19}$$

which is the equation sought.

Similarly at any point P, L_2 distant from the load end where

$l = L_2$

$$\dot{E} = \dot{E}$$

and $\dot{I} = \dot{I}$

$$\dot{E} = \dot{E}_r \cosh L_2\sqrt{zy_s} + \dot{I}_r\sqrt{\frac{z}{y_s}}\sinh L_2\sqrt{zy_s} \tag{20}$$

Similarly substituting equations (*16*) and (*17*) in equation (*15*)

$$\dot{I}_g = \sqrt{\frac{y_s}{z}}\ (A\cosh L\sqrt{zy_s} + A\sinh L\sqrt{zy_s} - B\cosh L\sqrt{zy_s} + B\sinh L\sqrt{zy_s})$$

$$= \sqrt{\frac{y_s}{z}}\ (\ (A-B)\cosh L\sqrt{zy_s} + (A+B)\sinh L\sqrt{zy_s}).$$ (21)

Then substituting the values of $(A+B)$ and $(A-B)$ from equations (*12*) and (*13*) in equation (*21*)

$$\dot{I}_s = \dot{I}_r \cosh L\sqrt{\overline{zy_s}} + \dot{E}_r\sqrt{\frac{\overline{y_s}}{z}} \sinh L\sqrt{\overline{zy_s}} \qquad (22)$$

Similarly at any point P, L_2 distant from the load end

$$l = L_2$$

$$\dot{E} = \dot{E}$$

and $$\dot{I} = \dot{I}.$$

By the same method as used in obtaining equation (*22*)

$$\dot{I} = \dot{I}_r \cosh L_2\sqrt{\overline{zy_s}} + E_r\sqrt{\frac{\overline{y_s}}{z}} \sinh L_2\sqrt{\overline{zy_s}} \qquad (23)$$

APPENDIX B

DERIVATION OF THE FORMULA FOR CALCULATION OF INDUCTIVE REACTANCES

In the following paragraphs is given the ordinary derivation of the formula for inductive reactance in open wire circuits.

The diagram shown in Figure 2 is a cross-sectional view of an electric circuit consisting of two conductors A and B, of radius r, spaced D units apart, each conductor carrying I amperes.

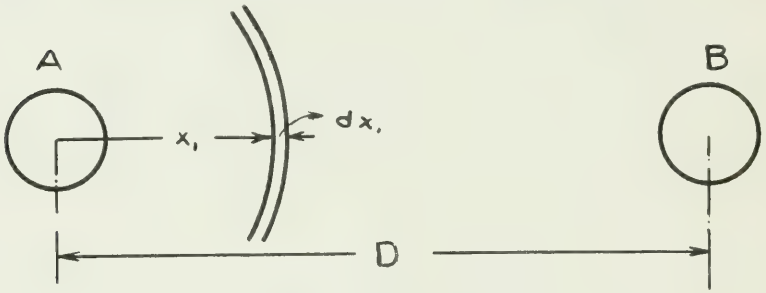

Fig. 2. Electromagnetic Field Surrounding a Conductor.

The fundamental law of the electromagnetic circuit is

$$Flux = \frac{Magnetomotive\ Force}{Reluctance}$$

$$\text{or } \phi = \frac{M.M.F.}{\mathfrak{R}}$$

Magnetomotive force varies directly with the number of turns, n, in the circuit and the value of the current in amperes, I,

$$\text{or } M.M.F. = \frac{4\ \pi\ n\ I}{10}$$

Reluctance varies directly with the length, l, of the flux path, inversely with the area, A, and depends on the permeability, μ, of the material through which the flux passes.

$$\text{or } \mathfrak{R} = \frac{l}{A\mu}$$

Substituting values for $M.M.F.$ and $\mathfrak{R}$ in the equation for flux

$$\phi = \frac{4\ \pi\ n\ I\ A\ \mu.}{10\ l}$$

Considering the line length one centimeter and an element dx_1 centimeters in width, x_1 centimeters from conductor A as shown in Figure 2.

$$d\phi = \frac{4\ \pi\ I\ \mu\ dx_1}{10\ .\ 2\ \pi\ x_1}$$

$$= \frac{I}{5}\ .\ \frac{dx_1}{x_1} \quad (\mu = 1\ for\ air)$$

Integrating $\phi = \frac{I}{5} \int_r^D \frac{dx_1}{x_1}$

or, $\phi = \frac{I}{5}\ log_e \frac{D}{r}$

ϕ is the flux outside the wire A due to the current in the wire A.

In addition to the flux outside the wire there is a certain amount of flux within the cross-section of the wire. In accounting for this last flux it is convenient to determine the amount of flux outside the wire that will produce the same effect as the flux within the wire. This may be done as follows:

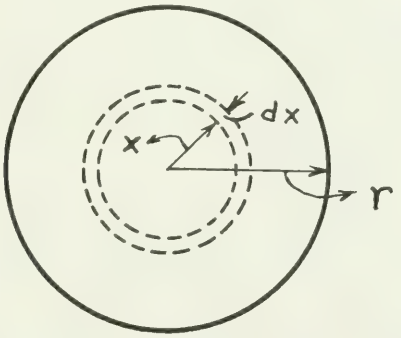

Fig. 3. Electromagnetic Field Within a Conductor.

As before

$$Flux = \frac{Magnetomotive\ Force,}{Reluctance}$$

$$\phi = \frac{M.M.F.,}{\mathfrak{R}}$$

$$\phi = \frac{4\ \pi\ n\ \ I\ \mu\ A.}{10\ l}$$

Considering a length of one centimeter of copper wire and the current per unit area as I'

$$d\phi_2 = \frac{4\ \pi\ .\ \pi\ x^2\ I^1\ .\ dx_2}{10\ .\ 2\ \pi\ x_2}$$

Simplifying $(\mu = 1)$

$$d\phi_2 = \frac{2}{10}\pi\ I^1\ x_2\ dx_2.$$

If $d\phi^1$ is the flux linking all of the fibers of the wire which will be equivalent to the flux $d\phi_2$ within the wire

then $$\frac{d\phi^1}{d\phi_2} = \frac{x_2^2}{r^2}$$

and, $$d\phi^1 = \frac{x^2}{r^2} d\phi_2$$

Substituting in this equation the value of $d\phi_2$

$$d\phi^1 = \frac{2\ \pi\ I^1\ x_2^3\ dx_2}{10\ r^2}$$

Integrating between the limits of 0 and r

$$\phi^1 = \frac{2\ \pi\ I^1}{10\ r^2} \int_0^r x_2^3\ dx_2,$$

$$\phi^1 = \frac{\pi\ r^2\ I^1}{20}$$

But $\pi\ r^2\ I^1$ is the area of the wire times the current per unit area and the product is the total current, I

Hence $\phi^1 = \frac{I}{20}$ is the equivalent flux within the wire.

The total effective flux is

$$\Phi = \phi + \phi^1,$$

Substituting the vauels of ϕ and ϕ^1 found above

$$\Phi = \frac{I}{20} + \frac{2I}{10}\ \log_\epsilon \frac{D}{r}$$

This value Φ is the flux per centimeter length of line. The coefficient of self inductance equals the flux linkages per unit of current.

$$\text{or, } L = \frac{\Phi}{I\ (10^8)}.$$

Substituting for the total flux Φ

$$L = \left(\frac{1}{20} + \frac{2}{10} \log_{\epsilon} \frac{D}{r}\right) 10^{-3}$$

where L is in henries per centimeter length of conductor

Reduced to a per mile basis and common logarithms the inductance in millihenries per mile of conductor is

$$L_m = (.0805 + .741 \log_{10} \frac{D}{r}).$$

The inductive reactance is

$$X = 2 \pi f L,$$

where f is the alternating current frequency and L is the inductance in henries. For a frequency of 60 cycles, substituting L_m for L.

$$x = 377 L_m 10^{-3}$$

or $x = (.03035 + .2793 \log_{10} \frac{D}{r})$ and is the reactance in ohms per mile.

APPENDIX C

DERIVATION OF THE FORMULA FOR THE CALCULATION OF SHUNTED ADMITTANCES

The formula for the shunted admittance may be derived as in the following paragraphs. In this formula the conductance to neutral is neglected as is also the inequality of electric charge in the conductors. The errors so introduced are very small for the range of wire sizes and spacings included in Tables 2 and 3.

The fundamental law of the electrostatic circuit is,

$$Dielectric\ Flux = \frac{Difference\ of\ Potential}{Elastance}$$

or $\psi = \frac{E}{S}$, *where* ψ *is the electrostatic flux and* S *the elastance in darafs*

The electostatic charge is equal to the capacity times the difference of potential,

$$\text{or } Q = C\,E,$$

As there are $4\ \pi$ lines of dielectric flux for each unit of charge,

$$\psi = 4\ \pi\ Q$$

Substituting for ψ and Q from the above equations

$$\frac{E}{S} = 4\ \pi\ C\ E,$$

$$\text{or } C = \frac{1}{4\ \pi\ S}.$$

Elastance varies directly with the length and inversely with the area of the electrostatic path. It depends on the material through which the flux passes as shown by the constant k in the formula

$$S = \frac{l}{Ak}$$

For air $k = 1$

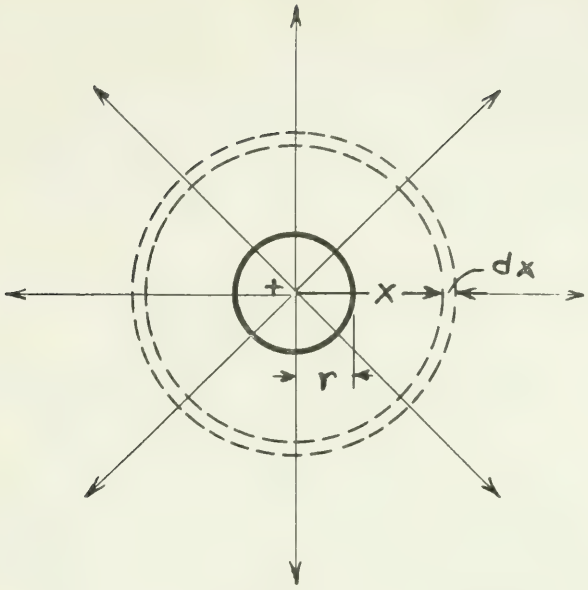

Fig. 4. Electrostatic Field Surrounding a Conductor.

Assume a charged wire in space, and therefore free from the proximity of disturbing conductors, then from Figure 4 the elastance of the differential section dx for one centimeter of wire length is,

$$ds = \frac{dx}{2\ \pi\ x} \qquad \text{(in air)}$$

Integrating between the limits x_1 and x

$$s = \frac{1}{2\pi} \int_{x_1}^{x_2} \frac{dx}{x},$$

or, $S = \frac{1}{2\pi}\ log_e\left(\frac{x_2}{x_1}\right)$ statdarafs per centimeter length of wire.

Reduced to the practical units of daraf and mile

$$S = 2.05\ log_{10}\left(\frac{x_2}{x_1}\right)\ 10^6 \text{ darafs per mile.}$$

The charging current for a condenser is $I = 2\ \pi\ f\ C\ E$, where f is the frequency, C the capacity n farads and E the voltage over the condenser.

Substituting the value $\frac{1}{4\pi S}$ for C in the charging current equation, $I=\frac{fE}{2S}$ or since $I=EY$, where Y is the admittance of the circuit the admittance of a condensive circuit in mhos is,

$$Y=\frac{f}{2S}.$$

The application of these fundamental relations to the parallel conductors of an open wire circuit is as follows:

Given: Wires A and B, as shown in Figure 2, D units of distance from center line to center line, each wire of radius r, and E volts difference of potential between wires.

Let ψA be the flux emitted from wire A per centimeter length
ψB be the flux emitted from wire B per centimeter length

Since all the flux leaving wire A is incident on wire B and the fluxes are in phase with the line to neutral voltages,

$$\psi_A+\psi_B=0.$$

Since the potentials of wires A and B are of opposite polarity with respect to the neutral or zero potential plane

$$\psi_A=\psi\ \underline{0^\circ}=\psi$$
$$\psi_B=\psi\ \underline{180^\circ}=-\psi$$

The absolute potential of A is (1) the elastance from wire A to wire B times the flux due to A, plus (2) the elastance from wire B to infinity times the flux due to wire A, plus, (3) the elastance from wire A to infinity times the flux due to wire B.

$$\text{Or, } E_A=\psi\left(\frac{1}{2\pi}\log_\epsilon\frac{D}{r}+\frac{1}{2\pi}\log_\epsilon\frac{x}{D}-\frac{1}{2\pi}\log_\epsilon\frac{x}{D}\right)$$

Similarly, the absolute potential of B is

$$E_B=-\psi\left(\frac{1}{2\pi}\log_\epsilon\frac{D}{r}+\frac{1}{2\pi}\log_\epsilon\frac{x}{D}-\frac{1}{2\pi}\log_\epsilon\frac{x}{D}\right)$$

Subtracting E_B from E_A gives the difference of potential between the two wires or E.

$$E_A-E_B=E=\frac{\psi}{\pi}\log_\epsilon\frac{D}{r}$$

The voltage to neutral is

$$\frac{E}{2} = \frac{\varphi}{2\pi} \log_\epsilon \frac{D.}{r}$$

Since elastance $S = \varphi E$ the elastance to neutral is

$$S = \frac{1}{2\pi} \log_\epsilon \frac{D}{r} \text{ statdarafs per centimeter length,}$$

$$\text{or, } S = 2.05 \left(\log_{10} \frac{D}{r}\right) 10^6 \text{ darafs per mile.}$$

For a frequency of 60 cycles the admittance to neutral per mile is,

$$y_s = \frac{f}{2S} = \frac{60}{2 \times 2.05 \log_{10} \frac{D}{r} \times 10^6} \text{ mhos.}$$

$$\text{or, } y_s = \frac{14.62 \times 10^{-6}}{\log_{10} \frac{D}{r}} \text{ mhos.}$$

APPENDIX D

SAMPLE CALCULATION

The following example illustrates the method used in calculating the tabular values.

Given:

Wire spacing = 120 inches. Wire size = 400,000 circular mils.

$$y_s = \frac{14.62 \times 10^{-6}}{\log_{10} \frac{D}{r}}$$

From Table 4 the radius of the cable $r = .364$ inches.

$$y_s = \frac{14.62 \times 10^{-6}}{\log_{10} \frac{120}{.364}}$$

$$y_s = 5.806 \times 10^{-6},$$

or in complex notation

$$y_s = j\ 5.806 \times 10^{-6}.$$

$$x = .03035 + .2793 \log_{10} \frac{D}{r},$$

$$= .03035 + .2793 \log_{10} \frac{120}{.364},$$

$$= .03035 + .7033,$$

$$= .7336.$$

$z = R + j\ x$ in which x is the inductive reactance per mile as found above and R is the resistance per mile from Table 4.

$$z = .1424 + j\ .7336$$

$$y = j\ 5.806 \times 10^{-6}$$

$$\text{and } zy_s = (-4.256 + j.8268) \times 10^{-6}$$

$$Arctan \frac{.8268}{-4.256} = arctan - .1942 = 169^\circ$$

$$\frac{169^\circ}{2} = 84^\circ 30'$$

$$cos\ \ 169^\circ = -.9816$$

$$sin\ \ 169^\circ = \ \ .1908$$

$$cos\ 84^\circ 30' = .0958$$

$$sin\ 84^\circ 30' = .9954$$

To extract the square root of a complex number, express the number in the polar form, take the square root of the magnitude and halve the angle. In polar form

$$zy_s = 4.3398 \times 10^{-6} / \underline{169°},$$

and, $$\sqrt{zy_s} = \sqrt{4.3398 \times 10^{-6}} \; / \underline{84°30'}.$$

Taking the square root and changing to rectangular form

$$\sqrt{zy_s} = (.1995 + j\ 2.076) \times 10^{-3}.$$

Also, $$\frac{z}{y_s} = (.1260 - j\ .0245) \times 10^{6}.$$

$$Arctan - \frac{.0245}{.1260} = arctan - .1942 = -11°$$

$$cos\ -11° = .9816$$

$$sin\ -11° = -.1908$$

$$cos\ -5°30' = .9954$$

$$sin\ -5°30' = -.0958$$

Following the same procedure as in the case of $\sqrt{zy_s}$

$$\sqrt{\frac{z}{y_s}} = \sqrt{.1283} \; / \underline{-5°30'} \times 10^{-3}.$$

and in rectangular form,

$$\sqrt{\frac{z}{y_s}} = (.3566 - j\ .0343) \times 10^{-3}.$$

LIST OF PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

CIRCULAR No. 1. Lightning Protection, by C. F. Harding, 1918.

BULLETIN No. 1. Flow of Water Through One and One-half Inch Pipe and Valves, by F. W. Greve, Jr., 1918.

BULLETIN No. 2. Electric Ranges, by C. W. Piper, 1919.

BULLETIN No. 3. Coefficients of Discharge of Sprinkler Nozzles, by F. W. Greve, Jr., and W. E. Stanley, 1919.

BULLETIN No. 4. Electric Driven Water Works in Indiana, by D. D. Ewing and G. C. Blalock, 1919.

BULLETIN No. 5. The Carburetion of Gasoline, by O. C. Berry and C. S. Kegerreis, 1920.

CIRCULAR No. 2. Preservative Treatment of Wood Poles, by R. V. Achatz, 1920.

BULLETIN No. 6. Sewage Disposal for Residences and Small Institutions, by R. B. Wiley, 1920.

BULLETIN No. 7. Tests of Road Materials of Indiana, by Testing Materials Staff, 1921.

BULLETIN No. 8. Flow of Water Through Spiral Riveted Steel Pipe, by F. W. Greve and R. R. Martin, 1921.

BULLETIN No. 9. The Production of Nitric Oxides and Ozone by High Voltage Electric Discharges, by K. B. McEachron, and R. H. George, 1922.

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BULLETIN No. 14. Tables of Transmission Line Constants, by D. D. Ewing, 1923.

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